

# A New 6-DOF Quadrotor Manipulation System: Design, Kinematics, Dynamics and Control

Mohamed Fanni and Ahmed Khalifa

**Abstract**—The research on aerial manipulation has been increased rapidly in recent years. In the previous work, a manipulator or a gripper is attached to the bottom of a quadrotor to facilitate the interaction with the environment. However, the previously introduced systems suffer from either limited end-effector DOFs or small payload capacity. In this article, we investigate a quadrotor with a 2-DOF manipulator that has a unique topology to enable the end-effector to track a desired 6-DOF trajectory with minimum possible actuators. Firstly, the proposed system is designed and modeled. However, such a system produces complexity to its inverse kinematics and control. Secondly, novel inverse kinematics analysis which requires a solution of complicated algebraic/differential equations is presented. Thus, an algorithm is developed to get an approximate solution of these equations, and its accuracy is verified. Thirdly, in order to solve the control problem, a simplified form of the nonholonomic constraints is utilized with a robust control technique. Fourthly, the controller stability is proved. Finally, experiments are conducted to validate the feasibility and the efficiency of the proposed system.

**Index Terms**—Aerial Manipulation, Quadrotor, Inverse Kinematics, Dynamic Modeling, Disturbance Observer.

## I. INTRODUCTION

Quadrotor system is one of the Unmanned Aerial Vehicles (UAVs) which offer possibilities of speed and access to regions that are otherwise inaccessible to ground robotic vehicles. Quadrotor vehicles possess certain essential characteristics, such as small size and cost, Vertical Take Off and Landing (VTOL), performing slow precise movements, and impressive maneuverability, which highlight their potential for use in vital applications [1]. However, most research on UAVs has typically been limited to monitoring and surveillance applications where the objectives are limited to “look” and “search” but “do not touch”. Due to their superior mobility, much interest is given to utilize them for aerial manipulation. Previous research on aerial manipulation can be divided into three approaches.

In the first approach, a gripper or a tool is installed at the bottom of an UAV to transport a payload or interact with existing structures. In [2]–[4], a quadrotor with a gripper is used for transporting blocks and to build structures. In [5], a force sensor is used to apply a normal force to a surface. Accordingly, not only the attitude of the payload/tool is restricted to the attitude of the UAV, but also the accessible range of the end-effector is confined due to the fixed

configuration of the gripper/tool with respect to the UAV body and blades. Consequently, the resulting aerial system has 4 DOFs; three translational DOFs and one rotational DOF (Yaw), i.e., the gripper/tool cannot possess pitch or roll rotation without moving horizontally. The second approach is to suspend a payload with cables. In [6], an adaptive controller is presented to avoid swing excitation of a payload. In [7]–[9], specific attitude and position of a payload is achieved using cables connected to one or three quadrotors. However, this approach has a drawback that the movement of the payload cannot be always regulated directly because manipulation is achieved using a cable which cannot always drive the motion of the payload as desired.

To overcome these limitations, a third approach is developed in which an aerial vehicle is equipped with a robotic manipulator that can actively interact with the environment. For example, in [10], a test bed including two 4-DOF robot arms and a crane emulating an aerial robot is proposed. By combining the mobility of the aerial vehicle with the versatility of a robotic manipulator, the utility of mobile manipulation can be maximized. When employing the robotic manipulator, the dynamics of the robotic manipulator are highly coupled with that of the aerial vehicle, which should be carefully considered in the controller design. Also, an aerial robot needs to tolerate the reaction forces from the interactions with the object or external environment. These reaction forces may affect the stability of an aerial vehicle significantly. Very few reports exist in the literature investigate the combination of aerial vehicle with robotic manipulator. Kinematic and dynamic models of the quadrotor combined with arbitrary multi-DOF robot arm are derived using the Euler-Lagrangian formalism in [11]. Based on that, simulation results using the Cartesian impedance control for the combined system with 3-DOF robotic arm is presented in [12]. In addition, the effects of manipulators, two 4-DOF arms, on the quadrotor are simulated based on the dynamic model which considers a quadrotor and robotic arms separately, treating the arms as the disturbance to the quadrotor control loop. In [13], a quadrotor with light-weight manipulators, three 2-DOF arms, are tested, although the movement of the manipulators is not explicitly considered during the design of the PID controller. In [14], an aerial manipulation using a quadrotor with a 2-DOF robotic arm is presented but with certain topology that disable the system from making arbitrary position and orientation of the end-effector. In this system, the axes of the manipulator joints are parallel to each other and parallel to one in-plane axis of the quadrotor. Thus, the system cannot achieve orientation around the second in-plane axis of the quadrotor without moving horizontally.

M. Fanni and A. Khalifa are with the Department of Mechatronics and Robotics Engineering, Egypt-Japan University of Science and Technology, New Borg El-Arab City, Alexandria, Egypt. e-mail: (mohamed.fanni@ejust.edu.eg) (ahmed.khalifa@ejust.edu.eg).

M. Fanni is on leave from Department of Production Engineering and Mechanical Design, Mansoura University, Mansoura, Egypt.

From the above discussion, the systems that use a gripper suffer from the limited allowable DOFs of the end-effector. The other systems have a manipulator with either two DOFs but in certain topology that disables the end-effector to track arbitrary 6-DOF trajectory, or more than two DOFs which decreases greatly the possible payload carried by the system.

In [15] and [16], the authors propose a new aerial manipulation system that consists of 2-link manipulator, with two revolute joints whose axes are perpendicular to each other and the axis of the first joint is parallel to one in-plane axis of the quadrotor. Thus, the end-effector is able to reach arbitrary position and orientation without moving horizontally. However, the proof that this system can perform any desired trajectory in the task space has not been achieved yet due to the resulted complications in the inverse kinematics and control. This system is similar to a serial manipulator with eight joints, two of them are passive.

This system opens new application area for robotics. Such applications are inspection, maintenance, firefighting, service robot in crowded cities to deliver light stuff such as post mails or quick meals, rescue operation, surveillance, demining, performing tasks in dangerous places, or transportation in remote places.

To sum up the main contributions of this article,

- New quadrotor-based aerial manipulator, which utilizes a 2-link robotic arm with unique topology such that the end-effector can track a desired 6-DOF trajectory, is designed. This system is non-redundant and use minimum possible number of actuators and links such that we can maximize the payload carried by the system and hence enlarge its practical use.
- Novel inverse kinematics analysis, which is based on complex set of nonlinear differential and algebraic equations, is presented and utilized to prove the end-effector ability to track arbitrary 6-DOF trajectory that lays within the capability of the used actuators.
- Design of a robust motion control scheme with rigorous stability analysis to track the desired 6-DOF trajectory.

In order to emulate a realistic setup, a simulation framework is developed to demonstrate the feasibility and the effectiveness of the approach in the presence of picking/placing a payload, uncertainties and sensor noise, and it is based on a real parameters obtained from an identification process of the considered system.

## II. DESCRIPTION AND DESIGN OF THE PROPOSED SYSTEM

3D CAD model of the proposed system is shown in Fig. 1. The system consists mainly of two parts; the quadrotor and the manipulator. Fig. 2 presents a sketch of the proposed system with the relevant frames which indicates the unique topology that permits the end-effector to achieve arbitrary pose. The frames are assumed to satisfy the Denavit-Hartenberg (DH) convention [17]. The manipulator has two revolute joints. The axis of the first revolute joint ( $z_0$ ), that is fixed to the quadrotor, is parallel to the body  $x$ -axis of the quadrotor (see Fig. 2). The axis of the second joint ( $z_1$ ) is perpendicular to the axis of the first joint and will be parallel to the body  $y$ -axis of

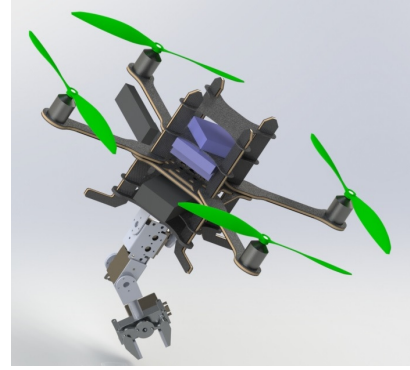


Figure 1: 3D CAD model of the proposed system

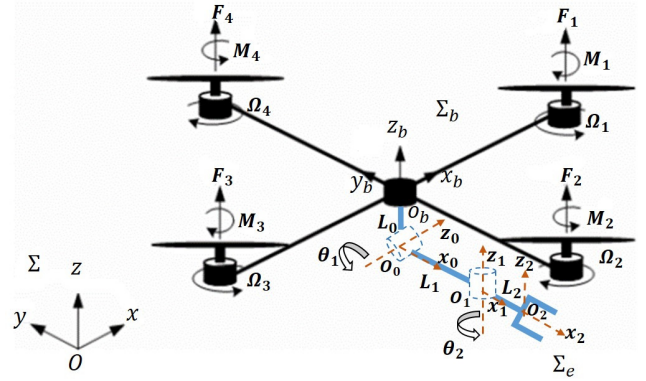


Figure 2: Schematic of Quadrotor Manipulation System with relevant frames

the quadrotor at home (extended) configuration. Thus, the pitching and rolling rotation of the end-effector is now possible independently from the horizontal motion of the quadrotor. Hence, with this new system, the capability of manipulating objects with arbitrary location and orientation is achieved. By this non-redundant system, the end-effector can achieve 6-DOF motion with minimum number of actuators/links which is an important factor in flight. The proposed system is distinguished from all other previous systems in the literature by having maximum mobility with minimum weight. The resulted complexity of the inverse kinematics and control are handled latter to prove the capability of the end-effector to track the reference 6-DOF trajectory.

The quadrotor components are selected such that it can carry an additional weight equals 500 g (larger than the total arm weight and the maximum payload). Asctec pelican quadrotor [18] is used as the quadrotor platform.

A lightweight manipulator that can carry a payload of 200 g and has a maximum reach of 22 cm is designed. The arm components are selected, purchased and assembled. Its weight is 200 g. It has a Flight Control Unit (FCU) "asctec Autopilot" as well as a modular structure which enables us to easily mount different components like computer boards, position sensors, and the robotic arm with its avionics. The FCU has an Inertial Measurement Unit (IMU) which provides the body accelerations, body angular velocities, magnetic compass, and the estimated attitude of the vehicle. The 6-DOF quadrotor

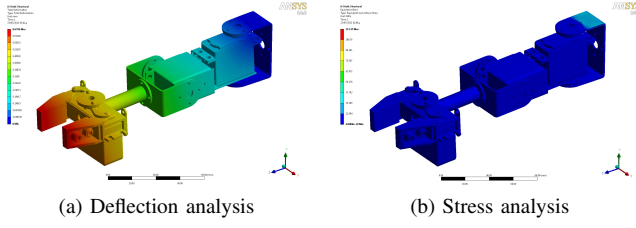


Figure 3: Manipulator's structure analysis

state estimation can be obtained via the onboard IMU and monocular vision data fusion [19].

The arm components are: Three DC motors; HS-422 (Max torque = 0.4 N.m) for the gripper, HS-5485HB (Max torque = 0.7 N.m) for joint 1, and HS-422 (Max torque = 0.4 N.m) for joint 2. Motor's Driver (SSC) is used as an interface between the main control unit and the motors. Wireless PS2 R/C is used to send commands to manipulator's motors remotely. The manipulator structure accessories are; Aluminum Tubing - 1.50 in diameter, Aluminum Multi-Purpose Servo Bracket, Aluminum Tubing Connector Hub, and Aluminum Long 'C' Servo Bracket with Ball Bearings [20]. An Arduino board (Mega 2560) is utilized as an interface between the low level peripherals (such as ultrasonic sensor, PS2 wireless receiver, and motor driver (SSC)), and the onboard computer. The angular position and velocity of the joints can be obtained from an encoder that is connected to each joint's motor.

The safety of this design, with respect to the strength and rigidity, is checked through finite element analysis using ANSYS software (see Fig. 3). A 250 g force is applied at the gripper end, while the arm end (i.e., point at which the arm is connect to the body center of the quadrotor) is fixed. From these figures, the maximum deflection is about 0.6 mm which is smaller than the allowable value which equals 1 mm. In addition, the maximum stress of the structure is 113 MPa which is smaller than the yield strength of aluminum alloy which is 270 MPa. Also, the bearings and gripper are selected to sustain the loads. Therefore, this design is safe.

### III. FORWARD KINEMATICS

In this section, position and velocity kinematics analysis are presented. Let  $\Sigma_b$ ,  $O_b$ -  $x_b$   $y_b$   $z_b$ , denotes the vehicle body-fixed reference frame with origin at the quadrotor center of mass, see Fig. 2. Its position with respect to the world-fixed inertial reference frame,  $\Sigma$ ,  $O$ -  $x$   $y$   $z$ , is given by the  $(3 \times 1)$  vector  $p_b = [x \ y \ z]^T$ , while its orientation is given by the rotation matrix  $R_b$ :

$$R_b = \begin{bmatrix} C_\psi C_\theta & S_\phi S_\theta C_\psi - S_\psi C_\phi & S_\psi S_\phi + C_\psi S_\theta C_\phi \\ S_\psi C_\theta & C_\psi C_\phi + S_\psi S_\theta S_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix}, \quad (1)$$

where  $\Phi_b = [\psi \ \theta \ \phi]^T$  is the triple  $ZYX$  yaw-pitch-roll angles. Note that  $C$  and  $S$  are short notations for  $\cos(\cdot)$  and  $\sin(\cdot)$ , respectively. Let us consider the frame  $\Sigma_e$ ,  $O_2$ -  $x_2$   $y_2$   $z_2$ , attached to the end-effector of the manipulator, see Fig. 2.

Thus, the position of  $\Sigma_e$  with respect to  $\Sigma$  is given by

$$p_e = p_b + R_b p_{eb}^b, \quad (2)$$

where the vector  $p_{eb}^b$  describes the position of  $\Sigma_e$  with respect to  $\Sigma_b$  expressed in  $\Sigma_b$ . The orientation of  $\Sigma_e$  can be defined by the rotation matrix

$$R_e = R_b R_e^b, \quad (3)$$

where  $R_e^b$  describes the orientation of  $\Sigma_e$  w.r.t  $\Sigma_b$ . The forward kinematics problem consists of determining the operational coordinates  $\chi_e = [x_e \ y_e \ z_e \ \psi_e \ \theta_e \ \phi_e]^T$ , as a function of the vehicle/joint space coordinates  $q = [x \ y \ z \ \psi \ \theta \ \phi \ \theta_1 \ \theta_2]^T$ . For solving the forward kinematics, the inputs are 8 variables,  $q$ , and the output are 6 variables,  $\chi_e$ , obtained from 6 algebraic equations. The end-effector position can be found from (2). Euler angles of the end-effector  $\Phi_e$  can be computed from the rotation matrix of  $R_e$ , from (3).

In the rest of this section, the velocity kinematics analysis is discussed. The linear velocity  $\dot{p}_e$  of  $\Sigma_e$  in the world-fixed frame is obtained by the differentiation of (2) as

$$\dot{p}_e = \dot{p}_b - Skew(R_b p_{eb}^b) \omega_b + R_b \dot{p}_{eb}^b, \quad (4)$$

where  $Skew(\cdot)$  is the  $(3 \times 3)$  skew-symmetric matrix operator [17], while  $\omega_b$  is the angular velocity of the quadrotor expressed in  $\Sigma$ . The angular velocity  $\omega_e$  of  $\Sigma_e$  is expressed as

$$\omega_e = \omega_b + R_b \omega_{eb}^b, \quad (5)$$

where  $\omega_{eb}^b$  is the angular velocity of the end-effector relative to  $\Sigma_b$  and is expressed in  $\Sigma_b$ .

Let  $\Theta = [\theta_1 \ \theta_2]^T$  be the  $(2 \times 1)$  vector of joint angles of the manipulator. The  $(6 \times 1)$  vector of the generalized velocity of the end-effector with respect to  $\Sigma_b$ ,  $v_{eb}^b = [\dot{p}_{eb}^b \ \omega_{eb}^b]^T$ , can be expressed in terms of the joint velocities  $\dot{\Theta}$  via the manipulator Jacobian,  $J_{eb}^b$ , [21] such that

$$v_{eb}^b = J_{eb}^b \dot{\Theta} \quad (6)$$

From (4) and (5), the generalized end-effector velocity,  $v_e = [\dot{p}_e^T \ \omega_e^T]^T$ , can be expressed as

$$v_e = J_b v_b + J_{eb} \dot{\Theta}, \quad (7)$$

where  $v_b = [\dot{p}_b^T \ \omega_b^T]^T$ ,  $J_b = \begin{bmatrix} I_3 & -Skew(R_b p_{eb}^b) \\ O_3 & I_3 \end{bmatrix}$ ,  $J_{eb} =$

$\begin{bmatrix} R_b & O_3 \\ O_3 & R_b \end{bmatrix} J_{eb}^b$ , where  $I_m$  and  $O_m$  denote  $(m \times m)$  identity and  $(m \times m)$  null matrices, respectively. If the attitude of the vehicle is expressed in terms of yaw-pitch-roll angles, then (7) will be

$$v_e = J_b Q_b \chi_b + J_{eb} \dot{\Theta}, \quad (8)$$

with  $\chi_b = \begin{bmatrix} p_b \\ \Phi_b \end{bmatrix}$ ,  $Q_b = \begin{bmatrix} I_3 & O_3 \\ O_3 & T_b \end{bmatrix}$ , where  $T_b$  describes the transformation matrix between the angular velocity  $\omega_b$  and the time derivative of Euler angles  $\dot{\Phi}_b$ , and it is given as

$$T_b(\Phi_b) = \begin{bmatrix} 0 & -S_\psi & C_\psi C_\theta \\ 0 & C_\psi & S_\psi C_\theta \\ 1 & 0 & -S_\theta \end{bmatrix}. \quad (9)$$

Since the vehicle is an under-actuated system, i.e., only 4 independent control inputs are available for the 6-DOF system, the position and the yaw angle are usually the controlled variables. Hence, it is worth to define  $\zeta = [x \ y \ z \ \psi \ \theta_1 \ \theta_2]^T$  as the controlled variables and  $\sigma_b = [\theta \ \phi]^T$  as the intermediate variables.

#### IV. DYNAMIC MODEL

The equations of motion of the proposed robot have been derived by us in details in [15]. Applying Newton Euler algorithm [21] to the manipulator considering that the link (with length  $L_0$ ) that is fixed to the quadrotor is the base link, one can get the equations of motion of the manipulator as well as the interaction forces and moments between the manipulator and the quadrotor. The effect of adding a payload to the manipulator will appear in the parameters of its end link, link 2, (e.g. mass, center of gravity, and inertia matrix). For the system structure, we assume:

**Assumption 1.** *The quadrotor body is rigid and symmetric. The manipulator links are rigid.*

The equations of motion of the manipulator are

$$M_1(q)\ddot{\theta}_1 + N_1(q, \dot{q}, \ddot{q}) = \tau_{m_1}, \quad (10)$$

$$M_2(q)\ddot{\theta}_2 + N_2(q, \dot{q}, \ddot{q}) = \tau_{m_2}, \quad (11)$$

where,  $\tau_{m_1}$  and  $\tau_{m_2}$  are the manipulator actuators' torques.  $M_1(q)$ ,  $M_2(q)$ ,  $N_1(q, \dot{q}, \ddot{q})$ , and  $N_2(q, \dot{q}, \ddot{q})$  are nonlinear terms and they are functions of the system states.

The Newton Euler method are used to find the equations of motion of the quadrotor after adding the forces/moments from the manipulator. They are given by (12-17).

$$m\ddot{x} = T(C_\psi S_\theta C_\phi + S_\psi S_\phi) + F_{m,q_x} \quad (12)$$

$$m\ddot{y} = T(S_\psi S_\theta C_\phi - C_\psi S_\phi) + F_{m,q_y} \quad (13)$$

$$m\ddot{z} = -mg + TC_\theta C_\phi + F_{m,q_z} \quad (14)$$

$$I_x\ddot{\phi} = \dot{\theta}\dot{\phi}(I_y - I_z) - I_r\dot{\theta}\bar{\Omega} + T_{a_1} + M_{m,q_\phi}^b \quad (15)$$

$$I_y\ddot{\theta} = \dot{\psi}\dot{\phi}(I_z - I_x) + I_r\dot{\phi}\bar{\Omega} + T_{a_2} + M_{m,q_\theta}^b \quad (16)$$

$$I_z\ddot{\psi} = \dot{\theta}\dot{\phi}(I_x - I_y) + T_{a_3} + M_{m,q_\psi}^b \quad (17)$$

where  $F_{m,q_x}$ ,  $F_{m,q_y}$ , and  $F_{m,q_z}$  are the interaction forces from the manipulator to the quadrotor in  $x, y$ , and  $z$  directions expressed in the inertial frame and  $M_{m,q_\phi}^b$ ,  $M_{m,q_\theta}^b$ , and  $M_{m,q_\psi}^b$  are the interaction moments from the manipulator to the quadrotor around  $x_b, y_b$ , and  $z_b$  directions.

The variables in (12-17) are defined as follows:  $m$  is the mass of the quadrotor. Each rotor  $j$  has angular velocity  $\Omega_j$  and it produces thrust force  $F_j$  and drag moment  $M_j$  which are given by

$$F_j = K_{f_j}\Omega_j^2, \quad (18)$$

$$M_j = K_{m_j}\Omega_j^2, \quad (19)$$

where  $K_{f_j}$  and  $K_{m_j}$  are the thrust and drag coefficients, respectively.

$T$  is the total thrust applied to the quadrotor from all four rotors, and it is given by

$$T = \sum_{j=1}^4 (F_j). \quad (20)$$

$T_{a_1}$ ,  $T_{a_2}$ , and  $T_{a_3}$  are the three input moments about the three body axes,  $x_b, y_b, z_b$ , and they are given as

$$T_{a_1} = d(F_4 - F_2), \quad (21)$$

$$T_{a_2} = d(F_3 - F_1), \quad (22)$$

$$T_{a_3} = -M_1 + M_2 - M_3 + M_4. \quad (23)$$

$d$  is the distance between the quadrotor center of mass and rotor rotational axis.  $\bar{\Omega}$  is given by

$$\bar{\Omega} = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4. \quad (24)$$

$I_r$  is the rotor inertia.  $I_f$  is the inertia matrix of the vehicle around its body-frame assuming that the vehicle is symmetric about  $x_b-$ ,  $y_b-$  and  $z_b-$ axis.

The dynamical model of the quadrotor-manipulator system can be written as follows

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + d_{ex} = Bu, \quad (25)$$

where  $M \in R^{8 \times 8}$  represents the symmetric and positive definite inertia matrix of the combined system,  $C \in R^{8 \times 8}$  is the matrix of Coriolis and centrifugal terms,  $G \in R^8$  is the vector of gravity terms,  $d_{ex} \in R^8$  is vector of the external disturbances,  $u = [F_1 \ F_2 \ F_3 \ F_4 \ \tau_{m_1} \ \tau_{m_2}]^T \in R^6$  is vector of the actuator inputs, and  $B = H(q) N(K_{f_j}, K_{m_j}, d)$  is the input matrix which is used to generate the body forces and moments from the actuator inputs. The control matrix,  $N$ , is given by

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ \gamma_1 & -\gamma_2 & \gamma_3 & -\gamma_4 & 0 & 0 \\ -d & 0 & d & 0 & 0 & 0 \\ 0 & -d & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (26)$$

where  $\gamma_j = K_{m_j}/K_{f_j}$ , and  $H \in R^{8 \times 8}$  is matrix that transforms body input forces to be expressed in  $\Sigma$  and is given by

$$H = \begin{bmatrix} R_b & O_3 & O_2 \\ O_3 & T_b^T R_b & O_2 \\ O_{2 \times 3} & O_{2 \times 3} & I_2 \end{bmatrix}. \quad (27)$$

From the equations of the translation dynamics part of (12-14), one can extract the expressions of the second order nonholonomic constraints as

$$\sin(\phi) - \frac{\ddot{x}_f S_\psi - \ddot{y}_f C_\psi}{\sqrt{\ddot{x}_f^2 + \ddot{y}_f^2 + \ddot{z}_f^2}} = 0, \quad (28)$$

$$\tan(\theta) - \frac{\ddot{x}_f C_\psi + \ddot{y}_f S_\psi}{\ddot{z}_f} = 0, \quad (29)$$

where  $\ddot{x}_f = \ddot{x} - \frac{F_{m,qx}}{m}$ ,  $\ddot{y}_f = \ddot{y} - \frac{F_{m,qy}}{m}$ , and  $\ddot{z}_f = \ddot{z} + g - \frac{F_{m,qz}}{m}$ .

It is to be noted that the force terms in the above equations are also function of the system states and their derivatives. Equations (28) and (29) can be solved for the desired trajectories of  $\phi$  and  $\theta$  through substituting by the desired trajectories of the other variables. These nonholonomic constraints will be utilized later to solve the inverse kinematics problem.

## V. INVERSE KINEMATICS

A task for the quadrotor manipulation system is usually specified in terms of a desired trajectory for the end-effector position,  $p_{e,r}(t)$ , and orientation,  $\Phi_{e,r}(t)$ . In this section, six algebraic kinematic equations relating  $\chi_e$  with  $q$  are derived.

However, to find the eight variables of  $q$  from the given six variables of  $\chi_e$ , we need additional two equations. These two equations are the nonholonomic constraints (28 and 29) which are differential equations. We solve these eight equation symbolically and reduce them to two differential equations in  $\sigma_b$ . It is found that the exact solution of them has very high computation cost which prevents its practical implementation.

For the desired task space trajectories, we assume:

**Assumption 2.** *The desired trajectories for the end-effector are bounded.*

Thus, we propose an algorithm to get an approximate solution to these eight algebraic/differential equations, see Fig. 4, as follows:

- 1) Specify the desired 6-DOF trajectory in the task space,  $\chi_{e,r}(t)$ .
- 2) Put  $i = 1$ ,  $t_1 = 0$ , and  $\Delta t =$  very small positive number (1 ms).
- 3) Put  $\sigma_b(t_0) = \dot{\sigma}_b(t_0) = \ddot{\sigma}_b(t_0) = [0 \ 0]^T$
- 4) At time  $t_i$ , obtain the values of  $\chi_{e,r}(t_i)$  and put  $\sigma_b(t_i) = \sigma_b(t_{i-1})$ ,  $\dot{\sigma}_b(t_i) = \dot{\sigma}_b(t_{i-1})$ , and  $\ddot{\sigma}_b(t_i) = \ddot{\sigma}_b(t_{i-1})$ .
- 5) Use the six inverse kinematics algebraic equations and get the unknowns  $\zeta_r(t_i)$ ,  $\dot{\zeta}_r(t_i)$ ,  $\ddot{\zeta}_r(t_i)$  which with  $\sigma_b(t_i)$ ,  $\dot{\sigma}_b(t_i)$ , and  $\ddot{\sigma}_b(t_i)$  constitute  $q(t_i)$ ,  $\dot{q}(t_i)$ , and  $\ddot{q}(t_i)$ .
- 6) In the equations of the nonholonomic constraints, we separate  $\sin(\phi)$  and  $\tan(\theta)$  in one side (left side) and the other terms in the other side (right side).
- 7) We substitute by  $q(t_i)$ ,  $\dot{q}(t_i)$ ,  $\ddot{q}(t_i)$  values in the right hand side of the nonholonomic constraints obtained from step 6, and obtain the new  $\sigma_b(t_i)$  from left hand side and by numerical differentiation, one can get the new  $\dot{\sigma}_b(t_i)$  and  $\ddot{\sigma}_b(t_i)$ .
- 8) Put  $i = i + 1$ ,  $t_i = t_{i-1} + \Delta t$ , and repeat steps 4 to 8 to obtain the reference trajectories of the controlled variables,  $\zeta_r(t_i)$ .

The validation of this algorithm will be checked at the end of this section. The six algebraic inverse kinematic equations are derived next.

Define the general form for the rotation matrix  $R_e$  as a function of end-effector variables  $\chi_e$ , as

$$R_e = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}. \quad (30)$$

Equating (3) and (30), an expression for the elements of  $R_e$ ,  $r_{ij}$ ;  $i, j = 1, 2, 3$ , can be found. According to the structure of  $R_e$  from (3), the inverse orientation is carried out first followed by inverse position. The inverse orientation has three cases based on the value of  $\theta_1$  which can be calculated from the element  $r_{33}$  as

$$C_\phi C_\theta C_\theta - S_\phi C_\theta S_{\theta_1} = r_{33}. \quad (31)$$

By rearranging (31) and solving the resulting equation for  $\theta_1$ , then

$$\theta_1 = 2 \arctan \left( \frac{-2b_1 \pm \sqrt{(2b_1)^2 - 4(-a_1 - r_{33})(a_1 - r_{33})}}{2(-a_1 - r_{33})} \right), \quad (32)$$

where  $a_1 = C_\phi C_\theta$  and  $b_1 = -S_\phi C_\theta$ .

The three cases are derived as follows:

**CASE 1:**  $\theta_1 \neq 0$  and  $\theta_1 \neq \pi$

Inspecting  $r_{13}$  and  $r_{23}$ , one can find value of  $\psi$  as

$$r_{13} = a_2 S_\psi + b_2 C_\psi, \quad (33)$$

$$r_{23} = c_2 S_\psi + d_2 C_\psi, \quad (34)$$

where  $a_2 = C_\phi S_{\theta_1} + S_\phi C_{\theta_1}$ ,  $b_2 = C_\phi S_\theta C_{\theta_1} - S_\phi S_\theta S_{\theta_1}$ ,  $c_2 = b_2$ , and  $d_2 = -a_2$ .

Solving (33) and (34), will give

$$S_\psi = \frac{r_{23}b_2 - r_{13}d_2}{-d_2a_2 + b_2c_2}, \quad (35)$$

$$C_\psi = \frac{r_{23}a_2 - r_{13}c_2}{d_2a_2 - b_2c_2}, \quad (36)$$

$$\psi = \text{atan2}(S_\psi, C_\psi). \quad (37)$$

By inspecting  $r_{32}$  and  $r_{31}$ ,  $\theta_2$  can be found as

$$r_{32} = a_3 S_{\theta_2} + b_3 C_{\theta_2}, \quad (38)$$

$$r_{31} = c_3 S_{\theta_2} + d_3 C_{\theta_2}, \quad (39)$$

where  $a_3 = C_\phi C_\theta S_{\theta_1} + C_\theta S_\phi C_{\theta_1}$ ,  $b_3 = -S_\theta$ ,  $c_3 = b_3$ , and  $d_3 = -a_3$ .

Solving (38) and (39), then

$$S_{\theta_2} = \frac{r_{31}b_3 - r_{32}d_3}{-d_3a_3 + b_3c_3}, \quad (40)$$

$$C_{\theta_2} = \frac{r_{31}a_3 - r_{32}c_3}{d_3a_3 - b_3c_3}, \quad (41)$$

$$\theta_2 = \text{atan2}(S_{\theta_2}, C_{\theta_2}). \quad (42)$$

**CASE 2:**  $\theta_1 = 0$

If  $\theta_1 = 0$ , then the sum  $\theta_2 + \psi$  can be determined. One can assume any value for  $\psi$  and get  $\theta_2$ . Therefore, there are infinity of solutions. By putting  $\psi = 0$ , the value of  $\theta_2$  can be determined as follows

Inspecting  $r_{11}$  and  $r_{12}$ ,  $\theta_2$  can be found as

$$r_{11} = a_4 S_{\theta_2} + b_4 C_{\theta_2}, \quad (43)$$

$$r_{12} = c_4 S_{\theta_2} + d_4 C_{\theta_2}, \quad (44)$$

where  $a_4 = C_{\theta}$ ,  $b_4 = -S_{\phi}S_{\theta}$ ,  $c_4 = -b_4$ , and  $d_4 = a_4$ . Solving (43) and (44), then

$$S_{\theta_2} = \frac{r_{12}b_4 - r_{11}d_4}{-d_4a_4 + b_4c_4}, \quad (45)$$

$$C_{\theta_2} = \frac{r_{12}a_4 - r_{11}c_4}{d_4a_4 - b_4c_4}, \quad (46)$$

$$\theta_2 = \text{atan2}(S_{\theta_2}, C_{\theta_2}). \quad (47)$$

**CASE 3:**  $\theta_1 = \pi$

Since  $S_{\theta_1} = 0$ , this is similar to case 2. However,  $\theta_2 - \psi$  can be determined and by choosing  $\psi = 0$ , an expression for  $\theta_2$  can be determined as follows:

Inspecting  $r_{11}$  and  $r_{12}$ ,  $\theta_2$  can be found as

$$r_{11} = a_5S_{\theta_2} + b_5C_{\theta_2}, \quad (48)$$

$$r_{12} = c_5S_{\theta_2} + d_5C_{\theta_2}, \quad (49)$$

where  $a_5 = a_4$ ,  $b_5 = -b_4$ ,  $c_5 = -b_5$ , and  $d_5 = a_5$ . Solving (48) and (49), then

$$S_{\theta_2} = \frac{r_{12}b_5 - r_{11}d_5}{-d_5a_5 + b_5c_5}, \quad (50)$$

$$C_{\theta_2} = \frac{r_{12}a_5 - r_{11}c_5}{d_5a_5 - b_5c_5}, \quad (51)$$

$$\theta_2 = \text{atan2}(S_{\theta_2}, C_{\theta_2}). \quad (52)$$

As shown above, there are four possible solutions for the rotational inverse kinematics problem provided that we put  $\psi = 0$  in case 2 and 3. When we start applying the above described algorithm in real time, we select the solution of  $\theta_1$  which coincide with the given configuration of the quadrotor manipulation system. After that, we continue with the one of the four solutions which produces this  $\theta_1$ .

Finally, the inverse position is determined from (2) as

$$p_b = p_e - R_b p_{eb}^b. \quad (53)$$

To validate the proposed approximate solution of the inverse kinematics, the desired task space trajectories are chosen to make a circular helix in position and quintic polynomial [17] for the orientation.

After obtaining the joint space variables from the proposed algorithm, as shown in Fig. 4, we apply the forward kinematics to find the actual task space trajectory.

The comparison between the actual and desired task space trajectories is shown in Fig. 5a from which one can recognize that the actual and desired trajectories are coincided. In Fig. 5b, another case study is investigated to show the capability of the proposed algorithm to deal with higher speed desired trajectories. In this study, the desired trajectories are chosen to be faster than that in Fig. 5a by ten times. These figures show that the actual and desired trajectories are coincided under both slower or faster trajectories, which ensures the validity of the proposed inverse kinematic algorithm, and proves that the proposed system has the ability to track arbitrary 6-DOF task space trajectory. With this result in mind, we propose a control algorithm in the next section to track the desired 6-DOF trajectories in the task space.

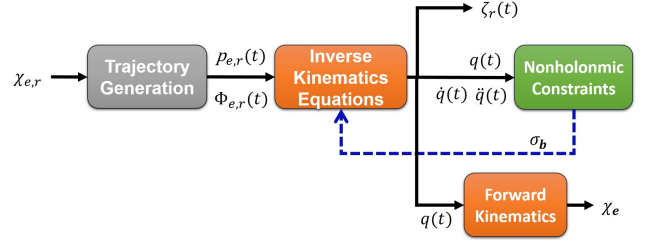


Figure 4: Verification of the inverse kinematic algorithm: Block diagram

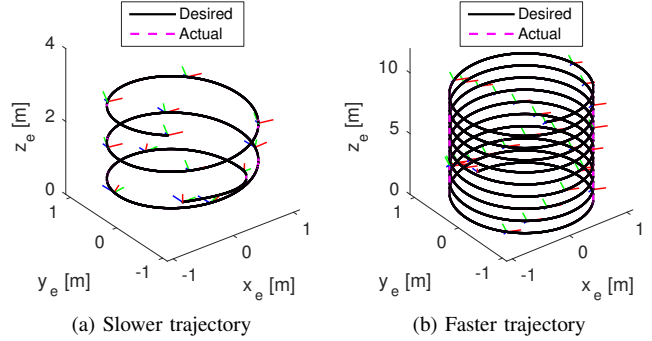


Figure 5: Verification of the inverse kinematic algorithm: End-effector 3D trajectory

## VI. CONTROL DESIGN

### A. Control Objectives

We target the design of the controller output,  $\tau$ , in order to satisfy the following objectives:

**Control Objective 1. (Robust Stability)** *The considered robotic system is stable and robust in the presence of the external disturbances and noises.*

**Control Objective 2. Minimizing the error of the 6-DOF task space trajectory tracking.**

To achieve these control objectives, we utilize a control technique based on both the Disturbance Observer (DOb) method and the above inverse kinematics analysis.

### B. DOb-based Controller

DOb-based controller is one of the most popular methods in the field of robust motion control due to its simplicity and computational efficiency. The authors in [22], [23] present the principles of DOb-based control system. In DOb-based robust motion control systems, internal and external disturbances are observed by DOb, and the robustness is simply achieved by feeding-back the estimated disturbances in an inner-loop. Another controller is designed in an outer loop so that the performance goals are achieved without considering internal and external disturbances. In [24]–[26], DOb-based control technique has been applied to robotic systems and showed efficient performance. The stability analysis of DOb is investigated in [27], [28], which show that the system stability is guaranteed with the DOb-based control.



A block diagram of the DOB controller is shown in Fig. 6 (The plant dynamics are highlighted by yellow box, while the DOB loop is highlighted by green box) for robotic system which will be utilized later to design robust control for the proposed system. It is well known that the linear accelerations and angular rates vehicle can be measured directly from the Inertial Measurement Unit (IMU). In addition, the angular velocities of the joints can be measured via an encoder. Therefore, two different DOB loops are used. One is based on the measured acceleration, while the other is based on the measured velocity.

In this figure,  $M_n = \begin{bmatrix} M_{n_a} & O_{3 \times 5} \\ O_{5 \times 3} & M_{n_v} \end{bmatrix} \in R^{8 \times 8}$  is the system nominal inertia matrix with  $M_{n_a} \in R^{3 \times 3}$  represents the nominal inertia for the translational coordinates,  $p_b$ , while  $M_{n_v} \in R^{5 \times 5}$  represents the nominal inertia for the rotational coordinates,  $\Phi_b$  and  $\Theta$ .  $\tau$  and  $\tau^{des}$  are the robot and desired inputs, respectively.  $Q(s) = \text{diag}([\frac{g_1}{s+g_1} \dots \frac{g_i}{s+g_i} \dots \frac{g_8}{s+g_8}]) \in R^{8 \times 8}$  is the matrix of the low pass filter of DOB,  $Q_a(s) = \text{diag}([\frac{g_1}{s+g_1} \dots \frac{g_3}{s+g_3}])$ , and  $Q_v(s) = \text{diag}([\frac{g_4}{s+g_4} \dots \frac{g_8}{s+g_8}])$ .  $P = \text{diag}([g_1 \dots g_i \dots g_8])$  with  $g_i$  is the bandwidth of the  $i^{th}$  variable of  $q$ , and  $P_v = \text{diag}([g_4 \dots g_i \dots g_8])$  for the velocity part.  $\tau^{dis}$  represents the system disturbances including the Coriolis, centrifugal and gravitational terms.  $\hat{\tau}^{dis} = [\hat{\tau}_a^{disT} \hat{\tau}_v^{disT}]^T$  (collected by the block called  $Mux$ , like multiplexer symbol in MATLAB) represents the system estimated disturbances.

If we apply the concept of disturbance observer to the proposed system, then the independent coordinate control is possible without considering coupling effect of other coordinates. The coupling terms such as centripetal and Coriolis and gravity terms are considered as disturbance and compensated by feed forward the estimated disturbance torque.

The system disturbance,  $\tau^{dis}$ , can be assumed as

$$\begin{aligned} \tau^{dis} &= (M(q) - M_n)\ddot{q} + \tau^d, \\ \tau^d &= C(q, \dot{q})\dot{q} + G(q) + d_{ex}. \end{aligned} \quad (54)$$

The control input,  $\tau$ , in Fig. 6 can be calculated as

$$\tau = M_n \ddot{q}^{des} + \hat{\tau}^{dis}, \quad (55)$$

where

$$\hat{\tau}^{dis} = Q(\tau - M_n \ddot{q}), \quad (56)$$

and the outer loop controller

$$\ddot{q}^{des} = \ddot{q}_r + K_P(q_r - q) + K_D(\dot{q}_r - \dot{q}), \quad (57)$$

where  $K_P$  and  $K_D \in R^{8 \times 8}$  are the proportional and derivative gains of the PD controller, receptively.  $q_r$ ,  $\dot{q}_r$ , and  $\ddot{q}_r$  are the references for linear/angular positions, velocities, and accelerations, respectively. By proper selection of  $K_P$  and  $K_D$ , one can tune the response of the the error dynamics as necessary. Thus, the control input,  $\tau$ , can be calculated as

$$\tau = M_n \ddot{q}^{des} + K_v e_v, \quad (58)$$

where  $K_v = P M_n = k_v I_8$ , and  $e_v = \dot{q}^{des} - \dot{q}$  with  $\dot{q}^{des}$  is the integration of  $\ddot{q}^{des}$  that is the output from the outer loop controller.

Fig. 7 presents a block diagram of the proposed motion control system based on the inverse kinematics analysis and on quadrotor/joint space-based control. The desired trajectories for the end-effector's position and orientation  $\chi_{e,r}(p_{e,r}(t)$  and  $\Phi_{e,r}(t))$  are fed to the inverse kinematics algorithm together with  $\sigma_{b,r}(t)$  from a simplified version of the nonholonomic constraints such that the desired vehicle/joint space trajectories  $\zeta_r(t)$  are obtained. After that, the controller block receives the desired trajectories and the feedback signals from the system and provides the control signal,  $\tau = Bu$ .

Since the position and the yaw angle are usually the controlled variables while pitch and roll angles are used as intermediate control inputs for horizontal positions control, the proposed control system consists from two DOB-based controllers; one for  $\zeta$  (with  $K_\zeta(s)$ ,  $M_{n_a}$ ,  $M_{nv_1}$ ,  $P_{v_1}$ ,  $Q_{v_1}$ , and  $Q_a$ ) and the other for  $\sigma_b$  (with  $K_\sigma(s)$ ,  $M_{nv_2}$ ,  $P_{v_2}$ ,  $Q_{v_2}$ ).

The desired values  $\sigma_{b,r}$  for the intermediate controller are obtained from the output of position controller,  $\tau_\zeta$ , through the relation

$$\sigma_{b,r} = \frac{1}{\tau_\zeta(3)} \begin{bmatrix} C_\psi & S_\psi \\ S_\psi & -C_\psi \end{bmatrix} \begin{bmatrix} \tau_\zeta(1) \\ \tau_\zeta(2) \end{bmatrix}, \quad (59)$$

which can be derived from (1) based on small angle approximation.

The desired value of the pitching and rolling,  $\sigma_{b,r}$ , is then fed back to the inverse kinematics algorithm.

The output of the two controllers,  $\tau_\zeta$  and  $\tau_\sigma$ , are mixed to generate the final control vector  $\tau$  which is converted to the forces/torques applied to quadrotor/manipulator by

$$u = B_6^{-1} \begin{bmatrix} \tau_\zeta(3,4) \\ \tau_\sigma \\ \tau_\zeta(5,6) \end{bmatrix}, \quad (60)$$

where  $B_6 \in R^{6 \times 6}$  is part of  $B$  matrix and it is given by  $B_6 = B(3 : 8, 1 : 6)$ .

**Theorem 1.** Consider the control system shown in Fig. 7, and described in (25, 55, and 57) with Assumptions 1–2. Then, as long as the following conditions;  $K_v > 0$ ,  $K_P > 0$ , and  $K_D > 0$ , are satisfied, the system is  $L_p$  input/output stable with respect to the pair  $(\delta, e_v)$ , and  $e \in L_p$  and  $\dot{e} \in L_p$  for all  $p \in [1, \infty)$  such that

$$\|e_v\|_p \leq \frac{1}{\gamma} + \sqrt{\frac{2}{\lambda_{min}} \left(\frac{2}{p\gamma}\right)^{\frac{1}{p}} \sqrt{V(0, e_v(0))}} \|\delta\|_p. \quad (61)$$

*Proof.* By substituting from the control law (58 and 57) into the system dynamics equation (25), one can get the following error dynamics

$$M(q)\dot{e}_v + C(q, \dot{q})e_v + K_v e_v = \delta, \quad (62)$$

and

$$\dot{e} + K_D e + K_P \int e dt = e_v, \quad (63)$$

where

$$\delta = \Delta M(q)\ddot{q}^{des} + C(q, \dot{q})\dot{q}^{des} + G(q) + d_{ex}, \quad (64)$$

and  $\Delta M(q) = M(q) - M_n$ . Assume the following Lyapunov function

$$V = \frac{1}{2} e_v^T M(q) e_v. \quad (65)$$

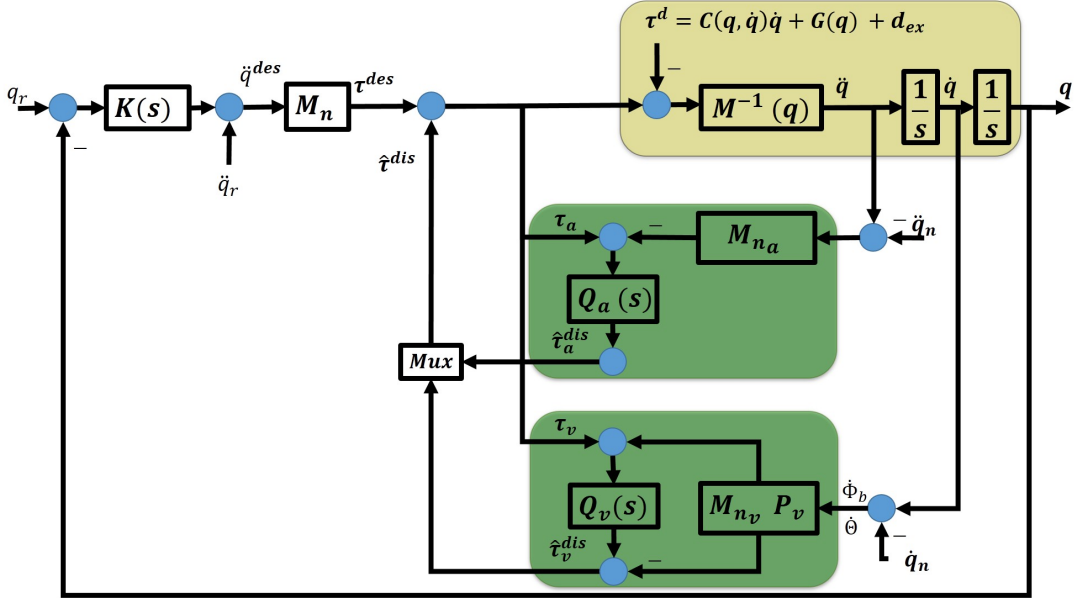


Figure 6: Block diagram of DOB-base controller

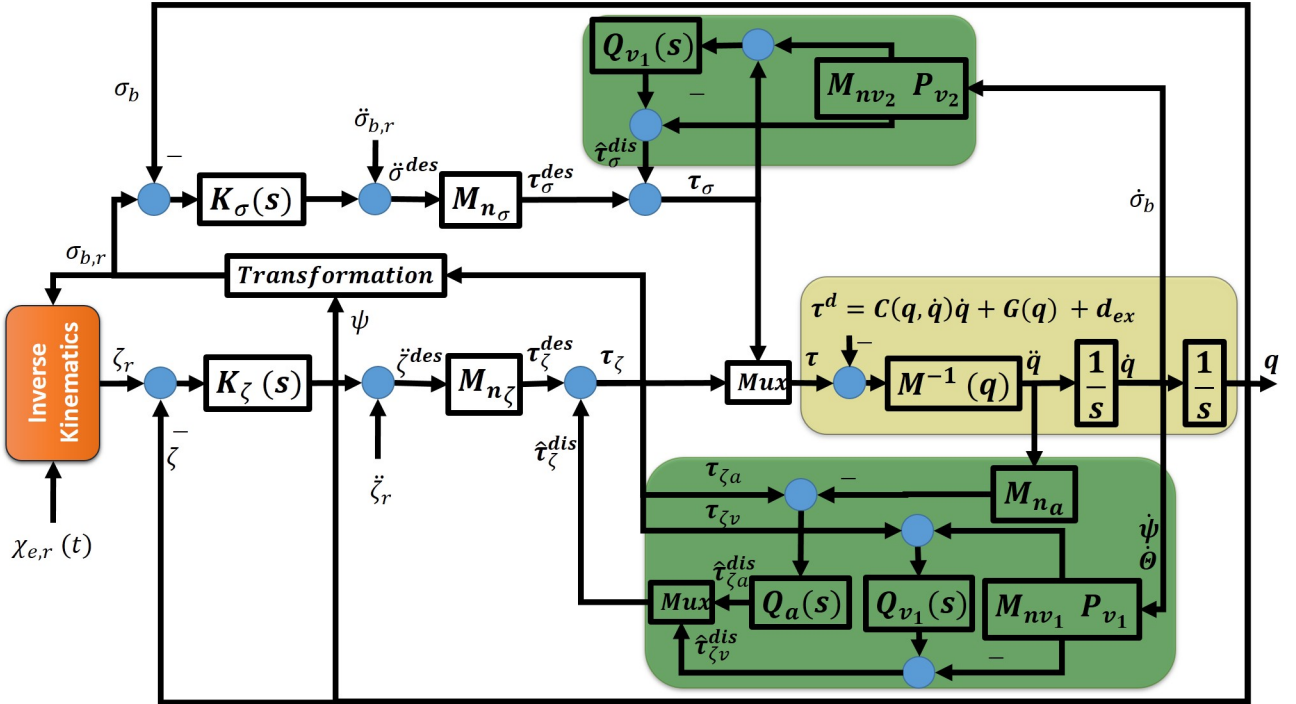


Figure 7: Detailed block diagram of the control system

The time derivative of this function is

$$\dot{V} = e_v^T M(q) \dot{e}_v + \frac{1}{2} e_v^T \dot{M}(q) e_v. \quad (66)$$

Substituting from (62), then (66) becomes

$$\dot{V} = e_v^T \delta - e_v^T K_v e_v + \frac{1}{2} e_v^T (\dot{M}(q) - 2C(q, \dot{q})) e_v. \quad (67)$$

The dynamic equation of motion (25) possesses several well known properties [17], [29].

**Property 1.**

$$\lambda_{min} \|\nu\|^2 \leq \nu^T M(q) \nu \leq \lambda_{max} \|\nu\|^2 \quad (68)$$

**Property 2.**

$$\nu^T (\dot{M}(q) - 2C(q, \dot{q})) \nu = 0 \quad (69)$$

where  $\dot{M}(q) - 2C(q, \dot{q})$  is a skew-symmetric matrix,  $\nu \in R^8$  represents a 8-dimensional vector, and  $\lambda_{min}$  and  $\lambda_{max}$  are positive real constants. Substituting from (69) into (67), then (67) will be

$$\dot{V} = e_v^T \delta - e_v^T K_v e_v. \quad (70)$$



From the robot dynamic property (68), one can find

$$\dot{V} \leq -\gamma V + \sqrt{\frac{2V}{\lambda_{\min}}} |\delta|, \quad (71)$$

where  $\gamma = \frac{2k_v}{\lambda_{\max}}$ . From the analysis presented in [30], [31], we complete the proof as following. Dividing (71) by  $V^{0.5} \neq 0$  will give

$$\frac{d}{dt}(V^{0.5}) + 0.5\gamma V^{0.5} \leq \sqrt{\frac{2}{\lambda_{\min}}} |\delta|, \quad (72)$$

By multiplying (72) by  $e^{-0.5\gamma t}$  and integrating, then we get

$$V^{0.5} \leq e^{-0.5\gamma t} V^{0.5}(0, e_v(0)) + V_c, \quad (73)$$

with

$$V_c = \frac{1}{\sqrt{2\lambda_{\min}}} \int_0^t e^{-0.5\gamma(t-l)} |\delta(l)| dl \quad (74)$$

We can rewrite  $V_c$  as

$$V_c = \frac{1}{\sqrt{2\lambda_{\min}}} (e^{-0.5\gamma t} * |\delta(t)|) \quad (75)$$

Taking the  $\|\cdot\|_p$ , gives

$$\|V_c\|_p = \frac{1}{\sqrt{2\lambda_{\min}}} \|e^{-0.5\gamma t} * \|\delta(t)\|\|_p \quad (76)$$

From Lemma 3 in [31], then (76) is

$$\|V_c\|_p = \frac{1}{\sqrt{2\lambda_{\min}}} \|e^{-0.5\gamma t}\|_1 \|\delta(t)\|_p \quad (77)$$

but  $\|e^{-0.5\gamma t}\|_1 = 0.5\gamma$ , so

$$\|V_c\|_p = 0.5\gamma \frac{1}{\sqrt{2\lambda_{\min}}} \|\delta(t)\|_p \quad (78)$$

Thus (73) is

$$\|V^{0.5}\|_p \leq \|e^{-0.5\gamma t} V^{0.5}(0, e_v(0))\|_p + 0.5\gamma \frac{1}{\sqrt{2\lambda_{\min}}} \|\delta(t)\|_p, \quad (79)$$

This expression (79) can be simplified to reach (61)

**Lemma 1.** ([32]) Let

$$e = H_{PD}(s)e_v \quad (80)$$

where  $H_{PD}(s)$  is an strictly proper and exponentially stable transfer function. Then  $e_v \in L_p$  implies that  $e \in L_p$  and  $\dot{e} \in L_p$ .

Form (63), the transfer function between  $e$  and  $e_v$  is proper exponentially stable as long as the parameters  $K_P$  and  $K_D$  are positive definite matrices. Thus, Lemma 1 implies that both  $e$  and  $\dot{e} \in L_p$ .  $\square$

*Remark 1.* Equation (71) indicates that the convergence rate increases proportionally with  $K_v$ . However, we have to take into considerations the constraints on  $g_i$  due to the sampling time.

*Remark 2.* It is noted from the quadrotor dynamics [33]–[35] that the response of the intermediate,  $\sigma$ , controller must be faster than that of the position control loop. However, as stated in the above analysis, the convergence rate of errors dynamics ( $e_v$  and  $e$ ) is directly proportional to the  $K_v$ ,  $K_P$ ,  $K_D$ . Therefore, we have to tune  $K_v$ ,  $K_P$ , and  $K_D$ , such that the response speed of  $\sigma_b$  is much faster than that of  $\zeta$ .

## VII. SIMULATION STUDY

In this section, the previously proposed control strategy is simulated in MATLAB/SIMULINK program for the considered aerial manipulation system.

### A. Simulation Environment

In order to make the simulation quite realistic, the following setup and assumptions have been made:

- The model has been identified on the basis of real data through experimental tests. The identified parameters are given in Table I.
- Linear and angular position and velocity of the quadrotor are available at rate of 1 KHz.
- The position and velocity of the manipulator joints are available at rate of 1 KHz.
- A normally distributed measurement noise, with mean of  $10^{-3}$  and standard deviation of  $5 \times 10^{-3}$ , has been added to the measured signals.
- The controller outputs are computed at a rate of 1 KHz.
- In order to test the robustness to the model uncertainties, a step disturbance is introduced, at instant 15 s, in the control matrix,  $N$ , (Actuators' losses), whose elements are assumed to be equal to 0.9 times their true values (i.e., 10% error).
- The end-effector has to pick a payload of value 200 g at instant 20 s and release it at 40 s.

Table I: System Parameters

Par.	Value	Unit	Par.	Value	Unit
$m$	1	kg	$L_2$	$85 \times 10^{-3}$	m
$d$	$223.5 \times 10^{-3}$	m	$m_0$	$30 \times 10^{-3}$	kg
$I_x$	$13.215 \times 10^{-3}$	N.m.s <sup>2</sup>	$m_1$	$55 \times 10^{-3}$	kg
$I_y$	$12.522 \times 10^{-3}$	N.m.s <sup>2</sup>	$m_2$	$112 \times 10^{-3}$	kg
$I_z$	$23.527 \times 10^{-3}$	N.m.s <sup>2</sup>	$I_r$	$33.216 \times 10^{-6}$	N.m.s <sup>2</sup>
$L_0$	$30 \times 10^{-3}$	m	$L_1$	$70 \times 10^{-3}$	m
$K_{F_1}$	$1.667 \times 10^{-5}$	kg.m.rad <sup>-2</sup>	$K_{F_2}$	$1.285 \times 10^{-5}$	kg.m.rad <sup>-2</sup>
$K_{F_3}$	$1.711 \times 10^{-5}$	kg.m.rad <sup>-2</sup>	$K_{F_4}$	$1.556 \times 10^{-5}$	kg.m.rad <sup>-2</sup>
$K_{M_1}$	$3.965 \times 10^{-7}$	kg.m <sup>2</sup> .rad <sup>-2</sup>	$K_{M_2}$	$2.847 \times 10^{-7}$	kg.m <sup>2</sup> .rad <sup>-2</sup>
$K_{M_3}$	$4.404 \times 10^{-7}$	kg.m <sup>2</sup> .rad <sup>-2</sup>	$K_{M_4}$	$3.170 \times 10^{-7}$	kg.m <sup>2</sup> .rad <sup>-2</sup>

### B. Results and Discussion

Tables II presents the controller parameter for the proposed control technique. Tuning of the controller parameters is carried out to maximize the speed of system response and minimize the tracking error. The reference trajectories for the end-effector are chosen such that the end-effector moves on a circular helix, while its orientation is fixed in a case and follows quintic polynomial trajectories in another case.

The simulation results are presented in Figs. 8 and 9. From these figures, it is possible to recognize that there are small oscillations after starting the system operation due to the time need to estimate the nonlinearities and disturbances. This time is about 4 s. After that period, the controller can quickly recover this error and provides good tracking of the

desired trajectories. Moreover, it is clear that the capability of the controller to recover, with fast response (about 1 s), the tracking error due to the presence of the payload and the uncertainty in system parameters. Furthermore, Fig. 10 indicates that the required actuators efforts, which are the required thrust force from each rotor and the torque from each manipulator's motor,  $u$ , are in the allowable limit. The maximum thrust force for each rotor is 6 N as obtained from the identification process, while the allowable input torque for the motor of joint 1 is 0.7 N.m and 0.4 N.m for that of joint 2 as stated in the motors' data sheet. Therefore, one can contend that the control objectives are achieved by using this motion control scheme.

Table II: DOB based controller parameters

Par.	Value	Par.	Value
$M_{n\zeta}$	$\text{diag}\{2, 2, 2, 0.5, 0.5, 0.5\}$	$M_{n\sigma}$	$\text{diag}\{0.5, 0.5\}$
$K_{P\zeta}$	$\text{diag}\{3, 3, 10, 3, 3, 3\}$	$K_{P\sigma}$	$\text{diag}\{10, 10\}$
$K_{D\zeta}$	$\text{diag}\{1.5, 1.5, 5, 3, 3, 3\}$	$K_{D\sigma}$	$\text{diag}\{7, 7\}$
$P$	$[10 \ 10 \ 10 \ 100 \ 100 \ 100 \ 100 \ 100]^T$		

## VIII. CONCLUSION

A new 6-DOF quadrotor-based aerial manipulator with minimum number of actuators is investigated. It provides solutions to the limitations found on the currently developed aerial manipulation systems by having maximum mobility with minimum weight. Description and design of the proposed system are introduced. Mathematical modeling are carried out. Proof that this system can perform any desired end-effector trajectory is done through analysis and numerical simulation of the complex inverse kinematics with nonholonomic constraints. A robust motion control scheme is designed and tested based on both the proposed inverse kinematics and the DOB technique in order to achieve 6-DOF task space trajectory tracking. Stability of the controller is analyzed. Simulation results enlighten the feasibility of the proposed system and the efficiency of the motion control scheme. As a future work, the proposed system will be tested experimentally.

## REFERENCES

- [1] S. Gupte, P. I. T. Mohandas, and J. M. Conrad, "A survey of quadrotor unmanned aerial vehicles," in *Southeastcon, 2012 Proceedings of IEEE*, pp. 1–6, IEEE, 2012.
- [2] D. Mellinger, Q. Lindsey, M. Shomin, and V. Kumar, "Design, modeling, estimation and control for aerial grasping and manipulation," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 2668–2673, IEEE, 2011.
- [3] Q. Lindsey, D. Mellinger, and V. Kumar, "Construction with quadrotor teams," *Autonomous Robots*, vol. 33, no. 3, pp. 323–336, 2012.
- [4] J. Willmann, F. Augugliaro, T. Cadalbert, R. D'Andrea, F. Gramazio, and M. Kohler, "Aerial robotic construction towards a new field of architectural research," *International journal of architectural computing*, vol. 10, no. 3, pp. 439–460, 2012.
- [5] A. Albers, S. Trautmann, T. Howard, T. A. Nguyen, M. Frietsch, and C. Sauter, "Semi-autonomous flying robot for physical interaction with environment," in *2010 IEEE Conference on Robotics Automation and Mechatronics (RAM)*, pp. 441–446, IEEE, 2010.
- [6] M. Bisgaard, A. la Cour-Harbo, and J. Dimon Bendtsen, "Adaptive control system for autonomous helicopter slung load operations," *Control Engineering Practice*, vol. 18, no. 7, pp. 800–811, 2010.

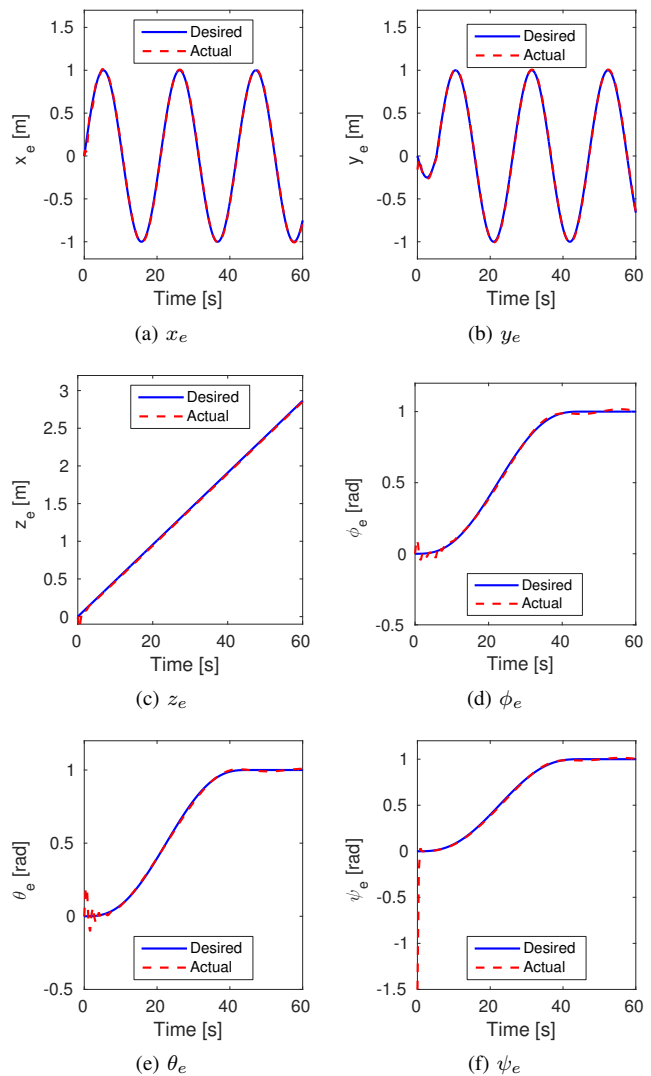


Figure 8: The actual response of the end-effector pose: a)  $x_e$ , b)  $y_e$ , c)  $z_e$ , d)  $\phi_e$ , e)  $\theta_e$ , and f)  $\psi_e$

- [7] N. Michael, J. Fink, and V. Kumar, "Cooperative manipulation and transportation with aerial robots," *Autonomous Robots*, vol. 30, no. 1, pp. 73–86, 2011.
- [8] F. A. Goodarzi, D. Lee, and T. Lee, "Geometric control of a quadrotor uav transporting a payload connected via flexible cable," *International Journal of Control, Automation and Systems*, vol. 13, no. 6, pp. 1486–1498, 2015.
- [9] F. A. Goodarzi and T. Lee, "Dynamics and control of quadrotor uavs transporting a rigid body connected via flexible cables," in *American Control Conference (ACC), 2015*, pp. 4677–4682, IEEE, 2015.
- [10] C. M. Korpela, T. W. Danko, and P. Y. Oh, "Mm-uav: Mobile manipulating unmanned aerial vehicle," *Journal of Intelligent & Robotic Systems*, vol. 65, no. 1–4, pp. 93–101, 2012.
- [11] V. Lippiello and F. Ruggiero, "Cartesian impedance control of a uav with a robotic arm," in *10th International IFAC Symposium on Robot Control*, pp. 704–709, 2012.
- [12] V. Lippiello and F. Ruggiero, "Exploiting redundancy in cartesian impedance control of uavs equipped with a robotic arm," in *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 3768–3773, IEEE, 2012.
- [13] M. Orsag, C. Korpela, and P. Oh, "Modeling and control of mm-uav: Mobile manipulating unmanned aerial vehicle," *Journal of Intelligent & Robotic Systems*, vol. 69, no. 1–4, pp. 227–240, 2013.
- [14] S. Kim, S. Choi, and H. J. Kim, "Aerial manipulation using a quadrotor with a two dof robotic arm," in *2013 IEEE/RSJ International Conference*

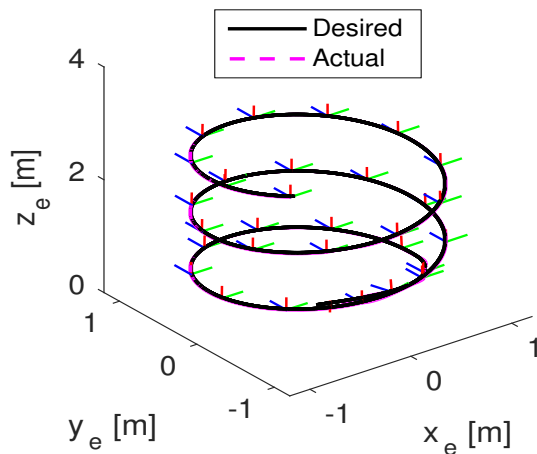


Figure 9: 3D trajectory of end-effector with fixed orientation (The marker represents the end-effector orientation; Green, Blue, and Red for x-,y, and z-axis)

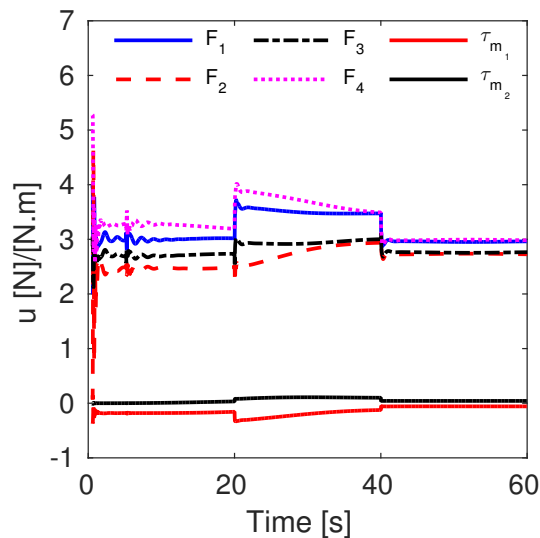


Figure 10: The required control efforts

- on *Intelligent Robots and Systems (IROS)*, pp. 4990–4995, IEEE, 2013.
- [15] A. Khalifa, M. Fanni, A. Ramadan, and A. Abo-Ismael, “Modeling and control of a new quadrotor manipulation system,” in *Proceedings of the IEEE/RAS International Conference on Innovative Engineering Systems*, pp. 109–114, IEEE, 2012.
- [16] A. Khalifa, M. Fanni, A. Ramadan, and A. Abo-Ismael, “Adaptive intelligent controller design for a new quadrotor manipulation system,” in *2013 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pp. 1666–1671, IEEE, 2013.
- [17] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot modeling and control*, vol. 3. Wiley New York, 2006.
- [18] *Asctec Pelican Quadrotor*, Oct. 2014. Available at <http://www.asctec.de/en/uav-uas-drone-products/asctec-pelican/>.
- [19] M. Achtelik, M. Achtelik, S. Weiss, and R. Siegwart, “Onboard imu and monocular vision based control for mavs in unknown in-and outdoor environments,” in *2011 IEEE international conference on Robotics and automation (ICRA)*, pp. 3056–3063, IEEE, 2011.
- [20] *LYNXMOTION*, Oct. 2014. Available at <http://www.lynxmotion.com/default.aspx>.
- [21] L.-W. Tsai, *Robot analysis: the mechanics of serial and parallel manipulators*. Wiley-Interscience, 1999.
- [22] B. K. Kim and W. K. Chung, “Performance tuning of robust motion controllers for high-accuracy positioning systems,” *IEEE/ASME Transactions on Mechatronics*, vol. 7, no. 4, pp. 500–514, 2002.

- [23] S. Li, J. Yang, W.-h. Chen, and X. Chen, *Disturbance observer-based control: methods and applications*. CRC Press, 2014.
- [24] E. Sariyildiz, H. Yu, K. Yu, and K. Ohnishi, “A nonlinear stability analysis for the robust position control problem of robot manipulators via disturbance observer,” in *2015 IEEE International Conference on Mechatronics (ICM)*, pp. 28–33, IEEE, 2015.
- [25] K. S. Eom, I. H. Suh, and W. K. Chung, “Disturbance observer based path tracking control of robot manipulator considering torque saturation,” *Mechatronics*, vol. 11, no. 3, pp. 325–343, 2001.
- [26] H.-T. Choi, S. Kim, J. Choi, Y. Lee, T.-J. Kim, and J.-W. Lee, “A simplified model based disturbance rejection control for highly accurate positioning of an underwater robot,” in *Oceans-St. John’s, 2014*, pp. 1–5, IEEE, 2014.
- [27] E. Sariyildiz and K. Ohnishi, “A guide to design disturbance observer based motion control systems,” in *2014 International Power Electronics Conference (IPEC-Hiroshima 2014-ECCE-ASIA)*, pp. 2483–2488, IEEE, 2014.
- [28] E. Sariyildiz and K. Ohnishi, “Stability and robustness of disturbance-observer-based motion control systems,” *IEEE Transactions on Industrial Electronics*, vol. 62, no. 1, pp. 414–422, 2015.
- [29] P. J. From, J. T. Gravdahl, and K. Y. Pettersen, *Vehicle-Manipulator Systems*. Springer, 2014.
- [30] N. Sadegh and R. Horowitz, “Stability and robustness analysis of a class of adaptive controllers for robotic manipulators,” *The International Journal of Robotics Research*, vol. 9, no. 3, pp. 74–92, 1990.
- [31] R. Bickel and M. Tomizuka, “Passivity-based versus disturbance observer based robot control: equivalence and stability,” *Journal of dynamic systems, measurement, and control*, vol. 121, no. 1, pp. 41–47, 1999.
- [32] P. A. Ioannou and J. Sun, *Robust adaptive control*. Courier Corporation, 2012.
- [33] M.-D. Hua, T. Hamel, P. Morin, and C. Samson, “Introduction to feedback control of underactuated vtolvehicles: A review of basic control design ideas and principles,” *Control Systems, IEEE*, vol. 33, no. 1, pp. 61–75, 2013.
- [34] A. Isidori, L. Marconi, and A. Serrani, *Robust autonomous guidance: an internal model approach*. Springer Science & Business Media, 2012.
- [35] Y.-C. Choi and H.-S. Ahn, “Nonlinear control of quadrotor for point tracking: Actual implementation and experimental tests,” *IEEE/ASME Transactions on Mechatronics*, 2015.



**Mohamed Fanni** received the B.E. and M.Sc. degrees in mechanical engineering from Faculty of Engineering of both Cairo University and Mansoura University, Egypt, in 1981 and 1986, respectively and the Ph.D. degree in engineering from Karlsruhe University, Germany, 1993. He is an Associate Professor with Innovation, Graduate School of Engineering Science, Egypt-Japan University of Science and Technology E-JUST, Alexandria, on leave from Production Engineering & Mechanical Design Department, Faculty of Engineering, Mansoura University, Egypt. His major research interests include robotics engineering, automatic control, and Mechanical Design. His current research focuses on Design & Control of Mechatronic Systems, Surgical Manipulators, Teleoperation systems and Flying/Walking Robots.



**Ahmed Khalifa** received the B.Sc. degree in Industrial Electronics and Control Engineering from Monofiya University, Egypt, in 2009, and the M.Sc. degree in Mechatronics and Robotics Engineering from Egypt-Japan University of Science and Technology, Alexandria, Egypt, in 2013. He is currently pursuing the Ph.D. degree in Mechatronics and Robotics Engineering at Egypt-Japan University of Science and Technology.